LETTERS TO THE EDITOR

To the Editor:

Recently Ho Nam Chang (1) reported a numerical method for the estimation of the effectiveness factor for the Michaelis-Menten type kinetics with high Thiele-Moduli. In the presence of external transport resistance the relevant equations are

$$\frac{1}{x^{s-1}}\frac{d}{dx}\left(x^{s-1}\frac{dy}{dx}\right) = \phi^2\frac{y}{\beta_1 + y}$$

(1) when $\beta_2 = 0$

$$Bc:\quad x = 1, \frac{dy}{dx} = Sh(1 - y) \qquad (2)$$

$$x = 0, \frac{dy}{dx} = 0 \qquad (3)$$

For higher values of ϕ, he suggested to solve equation (1) by a shooting technique between $x = \zeta$ and $x = 1$ with the value of $y(\zeta)$ calculated from the analytical solution for a first order kinetics. However, the value of ζ remains to be estimated by trial and error. Alternatively one point collocation can be applied to equations (1–3) with the concept of 'effective-reaction zone' (2). A trial function $y = a\xi^2$ is assumed to represent the concentration profile between $x = \lambda$ and $x = 1$ where ξ is given by

$$\xi = \frac{x - \lambda}{1 - \lambda} \qquad (4)$$

For large ϕ, $y(x = \lambda) = y(\xi = Q) = 0$ and $(dy/dx)_{x=\lambda} = 0$

Using the trial function along with equation (2)

$$a = \frac{1}{\left(1 + \dfrac{2}{1 - \lambda}\cdot\dfrac{1}{Sh}\right)} \qquad (5)$$

Introducing $y = a\xi^2$ and equation (5) in equation (1) we get a fourth order polynomial in λ. For a one point collocation ξ can be taken as the root of the Jacobi polynomial of order one (3). Therefore, λ can be obtained as the positive real root of the quartic with the constraint $0 < \lambda < 1$. Effectiveness factor η is then calculated as

$$\eta = \frac{s(1 + \beta_1)}{\phi^2}\left(\frac{2a}{1 - \lambda}\right) \qquad (6)$$

The values of η calculated in this way compares well with the values reported (1).

$Sh = 100$; $s = 3$; $\beta_2 = 0.0$; ϕ - 100

η values	$\beta_1 = 0.01$	$\beta_1 = 0.1$	$\beta_1 = 1.0$
(a) Ho Nam Chang (1)			
	0.215370 D-01	0.211400 D-01	0.277893 D-01
(b) One point collocation			
	0.217559 D-01	0.213558 D-01	0.270297 D-01
(c) Nine point collocation			
	0.215358 D-01	0.211169 D-01	0.275801 D-01
% Error (b-a/a) × 100			
	1.02	1.02	−2.73
% Error (b-c/c) × 100			
	1.02	1.13	−2.04

As can be seen the η values calculated by one point collocation are in good agreement with reported η values (1). Also the estimation of λ(ζ of Ref (1)) is explicit.

Literature cited

(1) Ho Nam Chang, "Numerical Calculation of Effectiveness Factors for the Michaelis-Menten Type Kinetics with High Thiele Moduli," *AIChE J.*, **28**, 1030 (1982).

(2) Paterson, W. R., and D. L. Cresswell, "A Simple Method for the Calculation of Effectiveness Factors," *Chem. Eng. Sci.*, *26*, 605 (1971).

(3) Villadsen, J., "Selected Approximation Methods for Chemical Engineering Problems," Danmarks Tekniske Hojskole (1970).

C. Lakshmanan
Department of Chemical Engineering
Monash University
Clayton Victoria, Australia

To the Editor:

This letter refers to two articles entitled "Analysis of particulate removal in Venturi Scrubbers—Effect of operating variables on performance" [*AIChE J* **27**, 981, (1981)] and "Analysis of particulate removal in Venturi Scrubbers—Role of heat and mass transfer" [*AIChE J* **28**, 31, 1982] by T. D. Placek and L. K. Peters. We observe the following discrepancies in these articles:

i) The Sautermean diameter for the drop size distribution reported by the authors, calculated using equation (3) in the first article is 101.68 μm while the authors have used a value of 175 μm.

ii) Equation (16) in the first article which defines α as "void fraction" actually represents the fraction of the cross-sectional area occupied by the droplets. The left hand side of this equation should be $(1 - \alpha)$ instead of α.

iii) For potential flow and for Stokes number in the range $0.055 \leq St < 0.12$, the authors have earlier reported (T. D. Placek and L. K. Peters (1980) and used in these articles, the following correlation for target efficiency.

$$E_1 = 1.96 + 1.8745\,ln\,(St) + 0.49539\,ln^2\,(St) + 0.06278\,ln^3\,(St) \qquad (1)$$

For the stipulated range of Stokes number, this equation predicts negative value for target efficiency which is unrealistic. It may be noted that the real root of the above equation is 0.2093.

iv) Similarly for viscous flow and for Stokes number less than 2, the reported target efficiency relationship

$$E_1 = 0.162615 + 0.13823\,ln\,(St)\quad St < 2 \qquad (2)$$

predicts negative value for target efficiency for Stokes number less than 0.308.

v) However, for viscous flow, in the expression for the interception effect,

$$F_4 = (F_2 + X)\,X \qquad (3)$$

the factor F_2 is negative for Stokes number less than 0.3416. Hence the correction factor F_4 is negative when $X < F_2$.

vi) In figure 3 of the first article, when the authors have compared their model prediction with the data of Ekman & Johnstone (1951), they have used the unit of liquid loading L as gal/1000 acf while in the nomenclature, they have reported the unit of L as m^3 liquid/m^3 gas.

vii) The numerical constant 0.01206 in the second term of the Nukiyama–Tanasawa equation reported by the authors (equation (2) in the first article), differs widely from the value of 0.916 (for the same set of units employed by the authors) reported by several other authors as mentioned below:

Many researchers have reported the Nukiyama–Tanasawa equation and there appears to be two different values for the above mentioned constant. The equation reported by Johnstone (1951), William Licht (1980), David Leith and Douglas Copper (1980), Roberts (1981), Strauss (1975) and Masters (1976) agrees with the value of 0.916 while L. P. Bayvel (1982) reports the value of 0.012. Goel and Hollands (1977) also appear to have used the same equation as the present authors.

Although the second term in the Nukiyama–Tanasawa equation is not very significant at low gas velocities and low liquid loadings, it becomes appreciable at high gas velocities and high

TABLE 1. DIMENSIONS OF THE SCRUBBER—SAME AS REPORTED BY THE AUTHORS FEED CONDITIONS OF GAS & LIQUID—BASE CASE OF THE AUTHORS. CASE—1 CORRESPONDS TO N-T EQUATION REPORTED BY THE AUTHORS. CASE—2 CORRESPONDS TO WIDELY QUOTED N-T EQUATION

Throat velocity m/s	Liquid Loading m^3L/m^3G	$\frac{C_f(\rho_p - \rho)d^2}{18\,\mu \times 10^6}$	Case—1			Case—2		
			$D_{32}\,\mu m$	St	NTU	$D_{32}\,\mu m$	St	NTU
60	0.0008	1.488	168.41	0.530	0.548	192.31	0.464	0.468
60	0.0008	5.952	168.41	2.120	2.986	192.31	1.857	2.760
60	0.0014	1.488	168.76	0.529	0.957	219.22	0.407	0.683
60	0.0014	5.952	168.76	2.116	5.219	219.22	1.629	4.417
60	0.0016	1.488	168.90	0.528	1.092	229.73	0.388	0.726
60	0.0016	5.952	168.90	2.114	5.962	229.73	1.554	4.875
120	0.0008	1.488	77.69	2.298	3.323	99.59	1.793	3.028
120	0.0008	5.952	77.69	9.193	7.119	99.59	7.172	6.795

liquid loadings. In table 1 we have compared the number of transfer units calculated using the Nukiyama–Tanasawa equation reported by the authors and the widely quoted Nukiyama–Tanasawa equation.

$$D_{32} = \frac{5 \times 10^{-3}}{V_{rel}} + 28.6 \times 10^{-6}\,(1000\,L)^{1.5} \quad (4)$$

We have used Runge-Kutta third order integration procedure with a step length of 0.001m and made the following modifications in the target efficiency correlations in the light of points (iii) and (iv) mentioned above.

Viscous flow

$$E_1 = 0.0\; St < 0.308 \quad (5)$$

$$E_1 = 0.162615 + 0.13823\, ln\,(St)\; 0.308 \leqslant St < 2 \quad (6)$$

Potential flow

$$E_1 = 0.65244 + 0.2530\, ln\,(St) + 0.069419\, ln^2\,(St) - 0.027269\, ln^3\,(St) \times 0.055 \leqslant St < 1.75 \quad (7)$$

The direct interception effect has been neglected. It is also pointed out, that since the authors have not given the values of the particle size and particle density, we used the same value for the factor

$$\frac{C_f(\rho_p - \rho)d^2}{18\,\mu}.$$

It is clear from the table, that the use of the Nukiyama–Tanasawa equation reported by the authors results in higher number of collection units, particularly at high liquid loadings.

Finally we feel that the plots of number of collection units as a function of the parameter $L\sqrt{ST}$ will be meaningful only when the particle size and particle density are also specified, because the collection efficiency is a function of both Stokes number and interception parameter.

LITERATURE CITED

Bayvel L. P., "The effect of polydispersity of drops on the efficiency of a venturi scrubber," *Trans. Inst. Chem. Engrs.*, **60**, 31 (1982).

David Leith and Douglas W. Cooper, "Venturi Scrubber Optimisation," *Atmospheric Environment*, **14**, 657 (1980).

Ekman F. O. and H. F. Johnstone, "Collection of Aerosols in a Venturi Scrubber," *Ind. Eng. Chem.*, **43**, 1358 (1951).

Hollands K. G. T. and K. C. Goel, "Optimum design of Venturi Scrubbers," *Atmospheric Environment*, **11**, 837 (1977).

Masters K., Spray Drying, George Godwin Ltd., London (1976).

Placek T. D. and L. K. Peters, "A hydrodynamic approach to particulate target efficiency in the presence of diffusiophoresis," *J. Aerosol. SCI.*, *II* 521 (1980).

Placek T. D. and L. K. Peters, "Analysis of particulate removal in Venturi Scrubbers—Effect of operating variables on performance," *AIChE J*, **27**, 981 (1981).

Placek T. D. and L. K. Peters, "Analysis of particulate removal in Venturi Scrubbers—Role of heat and mass transfer," *AIChE J*, **28**, 31 (1982).

Roberts D. B. and J. C. Hill, "Atomisation in a Ventrui Scrubber," *Chem. Eng. Commun*, **12**, 33 (1981).

Strauss W., Industrial gas cleaning, Pergamon Press (1975).

William Licht, Air Pollution Control Engineering, Marcel Dekker Inc., (1980).

V. S. Gururajan
A. Datta
R&D Department
Engineers India Limited
4, Sansad Marg
New Delhi-110 001
India.

Reply:

We thank V. S. Gururajan and A. Datta for their comments on the two papers that appeared in [*AIChE J.* (Placek and Peters, 1981, 1982)] and the one that was published in [*J. Aerosol Sci.* (Placek and Peters, 1980)].

Most of their comments have resulted from several typographical errors that were not caught from the galley proofs of the *J. Aerosol Sci.* paper. In that paper, we summarized our target efficiency analysis using strictly empirical correlations to represent the theoretical results. These were contained in Tables 1 and 2 of that paper, and the corrected constants are as follows.

i) Table 1, constant for third term in E_1 is 0.5̲9539 rather than 0.4̲9539.

ii) Table 2, constant for second term to calculate E_1 is 0.3̲18230 rather than 0.1̲38230. In addition, the limits for this equation are $0.60 < St < 2$ rather than simply $St > 2$, and $E_1 = 0$ for $St < 0.60$.

This should clarify any misconceptions raised by Gururajan and Datta's points (iii) and (iv). Moreover, relative to their point (v), the correlations, since they are strictly empirical, cannot be extended beyond the intended range of applicability. By referring to Figure 6 in Placek and Peters (1980), it can be readily noted that the correlations only apply to $St \gtrsim 0.5$.

In modeling scrubber performance, we have employed the Nukiyama–Tanasawa distribution, with modification, at the point of injection rather than at the throat, assuming no change in drop size distribution due to coalescence. This results in a different Sauter mean diameter, $\overline{D}_{32}$, for the following reasons. Since the injection site is in the converging section, the gas velocity is lower, resulting in a larger $\overline{D}_{32}$. Also, the distribution was modified to eliminate the large number of small drops (which do not significantly affect the overall collection). Finally, since the upper limit of the distribution is unbounded, a cut-off diameter was selected to represent the last size interval. With regard to their point (vii), the constant for the Nukiyama–Tanasawa equation should have read 0.9̲1206 rather than 0.0̲1206, which is of course consistent with the original reference and that most widely used.

We agree with Gururajan and Datta that other dimensionless parameters, in addition to $L\sqrt{St}$, could be important for correlating scrubber efficiency. Two parameters that they mention are particle density and size, which would enter non-dimensionally as ρ/ρ_p, and the interception parameter χ. However, most results and analyses have indicated that these have minor effects on venturi scrubber performance, and the traditional correlating parameter $L\sqrt{St}$ has worked quite well. In our calculations, the droplet motion was calculated using dimensionless variables, so that d_p and ρ_p enter through the drop and particle Stokes numbers and interception parameter. Finally, the abscissa of Figure 3 in the first article should be $L\sqrt{St} \times 10^4$.

The parameter α is the void fraction, and Gururajan and Datta correctly point out that the left hand side of Equation (16) should be $1 - \alpha$. This has been correctly used in all of the calculations.

Finally, a few comments should be made

about correctly using the empirical correlations in Placek and Peters (1980). As Tables 1 and 2 show, E_1 is not a target efficiency but simply one factor in calculating the target efficiency. Furthermore, it is generally accepted that there is a critical Stokes number, below which collection does not occur by pure impaction. This is about 0.55 for spherical collectors and spherical particles in the viscous flow regime and should be recognized. Therefore, it is ill-advised to extend collection by impaction much below the critical Stokes number.

We hope that these comments have clarified the "discrepancies," and appreciate the very thorough use of these papers by Gururajan and Datta.

T. D. Placek
Department of Chemical Engineering
Auburn University
Auburn, Alabama 36849

and

L. K. Peters
Department of Chemical Engineering
University of Kentucky
Lexington, Kentucky 40506

LITERATURE CITED

Placek, T. D. and L. K. Peters, "A Hydrodynamic Approach to Particle Target Efficiency in the Presence of Diffusiophoresis," *J. Aerosol Sci.*, *11*, 521 (1980).

Placek, T. D. and L. K. Peters, "Analysis of Particulate Removal in Venturi Scrubbers—Effect of Operating Variables on Performance," *AIChE J.*, *27*, 984 (1981).

Placek, T. D. and L. K. Peters, "Analysis of Particulate Removal in Venturi Scrubbers—Role of Heat and Mass Transfer," *AIChE J.*, *28*, 31 (1982).

To the Editor:

Marconi and Vatistas [*AIChE J.*, **29**, 513 (1983)] show, by Monte Carlo simulation, that the Danckwerts degree of segregation, J, for the random coalescence model of mixing is given by

$$J = \frac{1}{1 + I/2} \qquad (2)$$

where I is the coalescence rate parameter used by the authors.

This result has been obtained analytically, by integration of the population balance equations, by Verhoff (1969) and Komasawa *et al* (1971) and later used by Ross *et al* (1978) as the basis for measuring mixing rates. These authors did not, however, specifically identify their result as the Danckwerts "J" nor use it in the associated context.

Literature Cited

Komasawa, I., Morioka, S., Kuboi, R. & Otake, T. (1971) The method for measurement of interaction rates of dispersed phase in a continuous flow stirred tank, J. Chem. Soc. Japan, *4*, 319–324.

Marconi, P. F. & Vatistas, N. (1983) Degree of segregation and coalescence rate parameter in the random coalescence model for a stirred reactor [*AIChEJ.*, **29**, 513–516 (1983)].

Ross, S. L., Verhoff, F. H. & Curl, R. L. (1978) Droplet breakage and coalescence processes in an agitated dispersion. 2. Measurement and interpretation of mixing experiments. I&EC Fund. *17*, 101–108.

Verhoff, F. H. (1969) A study of the bivariate analysis of dispersed phase mixing. PhD Dissertation, University of Michigan, Ann Arbor, Mich.

Rane L. Curl
Department of Chemical Engineering
University of Michigan
Ann Arbor, Michigan 48109